**Course / Curriculum / Syllabus in compliance of NEP-2020**

**PhD 7 Level Courses**

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| **Sl. No.** | **Subject Code** | **Course Name** | **L** | **T** | **P** | **C** |
| 1. | MA7101/ MA7201 | Advanced Optimization Techniques | 3 | 0 | 0 | 3 |
| 2. | MA7102/ MA7202 | Algebra | 3 | 0 | 0 | 3 |
| 3. | MA7103/ MA7203 | An Introduction to Computational Commutative Algebra | 3 | 0 | 0 | 3 |
| 4. | MA7104/ MA7204 | Analysis I | 3 | 0 | 0 | 3 |
| 5. | MA7105/ MA7205 | Analysis-II | 3 | 0 | 0 | 3 |
| 6. | MA7106/ MA7206 | Differential Equations | 3 | 0 | 0 | 3 |
| 7. | MA7107/ MA7207 | Mathematical Control Theory | 3 | 0 | 0 | 3 |
| 8. | MA7108/ MA7208 | Probability Theory and Statistical Inference | 3 | 0 | 0 | 3 |
| 9. | MA7109/ MA7209 | Topology | 3 | 0 | 0 | 3 |

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| **Course Number** | MA7101 / MA7201 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Advanced Optimization Techniques |
| **Learning Mode** | Lectures |
| **Learning Objectives** | The objective of the course is to train student about the modeling of scalar and multiobjective nonlinear programming problems and various classical and numerical optimization techniques and algorithms to solve these problems |
| **Course Description** | Advanced Optimization Techniques, as a subject for postgraduate and PhD students, provides the knowledge of various models of nonlinear optimization problems and different algorithms to solve such problems with its applications in various problems arising in economics, science and engineering. |
| **Course Content** | Nonlinear programming: Convex sets and convex functions, their properties, convex programming problem, generalized convexity, Pseudo and Quasi convex functions, Invex functions and their properties, KKT conditions.  Goal Programming: Concept of Goal Programming, Model Formulation, Graphical solution method.  Separable programming. Geometric programming: Problems with positive coefficients up to one degree of difficulty, Generalized method for the positive and negative coefficients.  Search Techniques: Direct search and gradient methods, Unimodal functions, Fibonacci method, Golden Section method, Method of steepest descent, Newton-Raphson method, Conjugate gradient methods.  Dynamic Programming: Deterministic and Probabilistic Dynamic Programming, Discrete and continuous dynamic programming, simple illustrations.  Multiobjective Programming: Efficient solutions, Domination cones. |
| **Learning Outcome** | On successful completion of the course, students should be able to:  1. Understand the terminology and basic concepts of various kinds of nonlinear optimization problems  3. Develop the understanding about different solution methods to solve nonlinear Programing problems.  4. Apply and differentiate the need and importance of various algorithms to solve scalar and multiobjective optimization problems  5. Employ programming languages like MATLAB/Python to solve nonlinear programing problems  6. Model and solve several problems arising in science and engineering as a nonlinear optimization problem |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. Mokhtar S. Bazaaraa, Hanif D. Shirali and M. C. Shetty, Nonlinear Programming, Theory and Algorithms, John Wiley & Sons, New York (2004).
2. D.P. Bertsekas, Dynamic programming and Optimal Control, Athena Scientific, Belmont, 4th Edition, (2012).
3. S. Boyd and L. Vandenberghe: Convex Optimization, Cambridge University Press, New York, (2004).

**Reference Books:**

1. Edwin K. P. Chong and Stanislaw H. Zak: An Introduction to optimization, 4th Edition, John Wiley & Sons, New York, (2013).
2. D. G. Luenberger, Linear and Nonlinear Programming, Second Edition, Addison Wesley (2003).
3. DR. E. Steuer, Multi Criteria Optimization, Theory, Computation and Application, John Wiley and Sons, New York (1986).
4. Singiresu S. Rao, Engineering Optimization: Theory and Practice, John Wiley & Sons, (2020)
5. J. Nocedal and S. J. Wright, Numerical Optimization, Springer Verlag, (1999).

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| **Course Number** | MA7102 / MA7202 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Algebra |
| **Learning Mode** | Lectures |
| **Learning Objectives** | This course aims to help the students:  **(**1) gain a comprehensive understanding of algebraic structures as groups, rings, and vector spaces;  (2) well-equipped with basic concepts of Algebra which are prerequisites to the advanced courses;  (3) understanding of the advanced algebraic structures and their applications;  (4) help to research in Algebra, Fields and Galois Theory, Coding Theory, Cryptography, Homological Algebra, Noncommutative Algebra, Algebraic Geometry, and advanced topics on Analysis. |
| **Course Description** | It gives a foundation for further studies from the research perspective in the other courses of mathematics. Here, all basic topics of groups, rings, and linear algebra will be discussed. Besides the other examples of finite groups, this course includes Sylow’s theorems and their applications, Isomorphism theorems for groups and rings, Euclidean domain, UFD, quotient fields, and finite field extensions with several examples. The next part provides knowledge about the solution of the system of linear equations, vector spaces, linear transformation, singular value decompositions, inner product spaces and related concepts. |
| **Course Content** | Elementary set theory. Groups, subgroups, normal subgroups, homomorphisms, quotient groups, automorphisms, groups acting on sets, Sylow theorems and applications, finitely generated abelian groups. Examples: permutation groups, cyclic groups, dihedral groups, matrix groups. Basic properties of rings, units, ideals, homomorphisms, quotient rings, prime and maximal ideals, fields of fractions, Euclidean domains, principal ideal domains and unique factorization domains, polynomial rings. Elementary properties of finite field extensions and roots of polynomials, finite fields.  Vector spaces, Bases and dimensions, Change of bases and change of coordinates, Sums and direct sums, Quotient spaces. Linear transformations, Representation of linear transformations by matrices, The rank and nullity theorem, Dual spaces, Transposes of linear transformations. Trace and determinant, Eigenvalues and eigenvectors, Invariant subspaces, Direct-Sum decomposition, Cyclic subspaces and Annihilators, The minimal polynomial, The Jordan canonical form. Spectral theorem for normal operators, Quadratic forms. Singular value decomposition, polar decomposition. |
| **Learning Outcome** | On successful completion of the course, students should be able to:   1. Understand, apply, and analyze the notion of groups, rings, and ideals in related concepts required for advanced courses. 2. Familiar with the basic properties and examples of different notions of algebra and their generalization; 3. Able to decide what properties are satisfied by the given algebraic structure and under which conditions it can be applicable.   4. Able to use these concepts in the courses like, operator theory, algebraic coding theory, control theory and apply them for research purposes. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. D. Dummit and R. Foote, Abstract Algebra, 3rd edition, Wiley, 2004.
2. K. Hoffman and R. Kunze, Linear Algebra, Prentice-Hall, 1996.

**Reference Books:**

1. J. B. Fraleigh, A First Course in Abstract Algebra Paperback, Addison-wesley 1967.
2. Bernard Kolman, David Hill, Elementary Linear Algebra with Applications, 9th Edition, Pearson Education, 2007.
3. S. Lang, Linear Algebra, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1989.
4. S. Axler, Linear Algebra Done Right, 2nd Edition, UTM Series, Springer, 1997.
5. M. Artin, Algebra, Prentice Hall, 1994.

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| **Course Number** | MA7103 / MA7203 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | An Introduction to Computational Commutative Algebra |
| **Learning Mode** | Lectures |
| **Learning Objectives** | To expose students with the basic computational techniques in Commutative Algebra and its applications in some classical problems |
| **Course Description** | This course covers the classical theory of Grobner basis, Elimination theory and some of its first applications. |
| **Course Content** | Ring, Ideals, Ring homorphisms, polynomial rings, Unique factorization, polynomials and affine space, affine varieties, parametrization of affine varieties, monomial ordering: Lexicographic order, graded lex order, graded rev lex order, inverse lexicographic order etc, division algorithm for polynomials in n variables, monomial ideals, Dickson’s Lemma, Hilbert basis theorem, Grobner bases and its properties, Buchberger’s algorithm, reduced Grobner basis,  Applications of Grobner basis: Ideal description problem, Ideal membership problem, Solving polynomial equations, Implicitization problem, integer programming problem.  The elimination and extension theorems, Implicitization, Grobner basis and the extension theorem, Resultants and the extension theorem, |
| **Learning Outcome** | Students will learn the basic theory of Grobner basis, Hilbert basis theorem, a division algorithm for polynomials in n variables, elimination theory and extension theorems etc. Students will be exposed to various applications of Grobner basis in engineering and math problems. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. David A Cox, John Little and Donal O’Shea, , Ideals, Varieties and Algorithms, An introduction to computational Algebraic Geometry and Commutative Algebra, Fourth Addition, Springer Undergraduate texts in Mathematics
2. [Martin Kreuzer](https://link.springer.com/book/10.1007/978-3-540-70628-1#author-0-0)  and  [Lorenzo Robbiano](https://link.springer.com/book/10.1007/978-3-540-70628-1#author-0-1), Computational Commutative Algebra 1, first edition, Springer Berlin, Heidelberg

**Reference Books:**

1. David Eisenbud, Commutative Algebra with a view towards Algebraic Geometry, Springer-Verlag New York (1995).
2. [David S. Dummit](https://www.amazon.in/s/ref=dp_byline_sr_book_1?ie=UTF8&field-author=David+S.+Dummit&search-alias=stripbooks) and [Richard M. Foote](https://www.amazon.in/s/ref=dp_byline_sr_book_2?ie=UTF8&field-author=Richard+M.+Foote&search-alias=stripbooks), Abstract Algebra, third edition, Wiley Publication, 2011.

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| **Course Number** | MA7104 / MA7204 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Analysis I |
| **Learning Mode** | Lectures |
| **Learning Objectives** | To learn basic techniques and theorems of analysis which explains the intimate connections between its various branches. |
| **Course Description** | This course unifies real analysis and complex analysis and some of the basic ideas from functional analysis. |
| **Course Content** | Positive Borel, Complex Measures, Differentiation, Integration on Product Spaces, Elementary Properties of Holomorphic Functions, Harmonic Functions, The Maximum Modulus Principle, Approximation by Rational Functions, Conformal Mapping, Zeros of Holomorphic Functions, Analytic Continuation, Hp-Spaces, Elementary Theory of Banach Algebras, Holomorphic Fourier Transforms, Uniform Approximation by Polynomials.  **Prerequisite:** The prerequisite for this course is a good course in advanced calculus (set theoretic manipulations, metric spaces, uniform continuity, and uniform convergence). |
| **Learning Outcome** | Students should be able to understand the interconnection between various branches. |
| **Assessment Method** | Quiz /Assignment / Project / Presentations / MSE / ESE |

**Text Books:**

1. Walter Rudin: Real and Complex Analysis Third Edition, McGraw HiII Book Company.

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| **Course Number** | MA7105 / MA7205 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Analysis II |
| **Learning Mode** | Lectures |
| **Learning Objectives** | The main objectives of this course are to introduce advanced topics in Topology and Functional Analysis to the students.  They will learn to understand various spaces through their properties, mainly connectedness and compactness. Various important theorems associated with these concepts will be discussed in detail along with their applications.  Students will also understand the important consequences of continuous linear maps via theorems. Two new concepts of convergence, namely, weak and weak\* convergence will be introduced and their applications in solving problems will be highlighted. Various properties of compact operators will be discussed. |
| **Course Description** | This course covers advance topological concepts like one point compactification, local connectedness, local compactness etc. Theorems associated with these concepts will be discussed. Various countability and separation axioms are discussed in detail. Famous theorems namely, Urysohn’s Lemma, Urysohn’s metrization theorem, Tietze extension theorm, Tychonoff’s theorem will be discussed thoroughly with proofs.  This course also includes topics in functional analysis. Various results related to bounded linear operators, compact operators shall be discussed. Convergence in Banach spaces, reflexive spaces will be covered. Various operators on Hilbert spaces and their properties and applications will be discussed in detail. Spectral theorem for compact self-adjoint operators and bounded self-adjoint operators will be thoroughly discussed. |
| **Course Content** | Topology: Topological spaces, Basis for a topology, Limit points and closure of a set, Continuous and open maps, Homeomorphisms, Subspace topology, Product and quotient topology. Connected and locally connected spaces, Path connectedness, Components and path components, Compact and locally compact spaces, One point compactification. Countability axioms, Separation axioms, Urysohn’s Lemma, Urysohn’s metrization theorem, Tietze extension theorem, Tychonoff’’s theorem, Completely Regular Spaces, Stone-Cech Compactification.  Functional Analysis: Banach spaces, Continuity of linear maps, Hahn-Banach theorem, Open mapping and closed graph theorems, Uniform boundedness principle. Duals and Transposes. Compact operators and their spectra. Weak and Weak\* convergence, Reflexivity. Hilbert spaces, Bounded operators on Hilbert spaces. Adjoint operators, Normal, Unitary, Self-adjoint operators and their spectra. Spectral theorem for compact self-adjoint operators, statement of spectral theorem for bounded self-adjoint operators. |
| **Learning Outcome** | Upon completion of this course, the student should be able to  1. Understand various topological concepts such as compactness, connectedness and the theorems related to these concepts for various spaces.  2. Understand the separation and countability axioms and differentiate between certain properties of topological spaces.  3. Apply the theoretical concepts of topology to real world problems/research problems.  4. Gain knowledge on various central theorems in Functional Analysis such as the Hahn Banach Theorem, open mapping theorem, closed graph theorem.  5. Understand the applications of the spectral theorem for different operators. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. James R. Munkres, Topology, 2nd Edition, Prentice Hall, 1999.
2. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley, 1989.

**Reference Books:**

1. J. L. Kelley, General Topology, Springer International Edition, Indian Reprint, 2005.
2. M. A. Armstrong, Basic Toplogy, Springer- Verlag, 1997
3. M. Thamban Nair, Functional Analysis: A First Course, Prentice Hall of India, 2002.
4. G. F. Simmons, Introduction to Topology and Modern Analysis, Wiley, 2003.
5. J. B. Conway, A Course in Functional Analysis, GTM Series, Springer, 1990.

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| **Course Number** | MA7106 / MA7206 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Differential Equations |
| **Learning Mode** | Lectures |
| **Learning Objectives** | To provide understanding of ordinary and partial differential equations, their solutions. |
| **Course Description** | The differential equations arise in various real world problems. These are categorized as ordinary differential equations or partial differential equations. This course introduces not only techniques to solve these equations but also provides systematic understanding of methods of the solutions. |
| **Course Content** | Ordinary Differential Equations: First Order ODE y'=f(x,y)-geometrical Interpretation of solution, Equations reducible to separable form, Exact Equations, Integrating factor, Linear Equations, Orthogonal trajectories, Picard’s Theorem for IVP and Picard’s iteration method, Euler’ Method, Improved Euler’s Method, Elementary types of equations. F(x,y,y') =0; not solved for derivative.  Second Order Linear Differential equations: Fundamental system of solutions and general solution of homogeneous equation. Use of Known solution to find another, Existence and uniqueness of solution of IVP, Wronskian and general solution of non-homogeneous equations. Euler-Cauchy Equation, extensions of the results to higher order linear equations, Power Series Method application to Legendre Eqn., Legendre Polynomials, Frobenious Method, Bessel equation, Properties of Bessel functions, Sturm-Liouville BVPs, Orthogonal functions, Sturm comparison Theorem. Systems of Linear ODEs, Reduction of higher order linear ODEs to first order linear systems, Stability of linear systems.  Transforms: Fourier Series, Fourier transform and Laplace Transform. Solving Differential Equations using Transform methods.  Partial Differential Equations: Introduction to PDE, basic concepts, Linear and quasilinear first order PDE, Cauchy-Kowalewski theorem, second order PDE and classification of second order semi-linear PDE (Canonical form), D’ Alemberts formula and Duhamel’s principle for one dimensional wave equation, Laplace’s and Poisson’s equations, Maximum principle with application, Fourier Method for IBV problem for wave and heat equation, rectangular region, Fourier method for Laplace’s equation in three dimensions. |
| **Learning Outcome** | On successful completion of the course, students should be able to:  1. Understand the fundamentals of differential equations.  2. Grasp the concept of existence and uniqueness of solution of various types of differential equations.  3. Comprehend ideas of transforms and other related tools used for solving differential equations.  4. Understand not only what is a solution of the differential equation but also how the solution arise and what are basic properties of the solutions. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. G. Birkhoff and G. C. Rota, Ordinary Differential Equations, 4th Edition, Wiley Singapore Edition, 2003.
2. I. N. Sneddon, Elements of Partial Differential Equations, Dover, 2006.

**Reference Books:**

1. E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, Tata McGraw Hill, 1984.
2. R. Haberman, Applied Partial Differential Equations, 4th Edition, Prentice Hall, 2003.
3. L. Perko, Differential Equations and Dynamical Systems, Springer – Verlag, 2006

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| **Course Number** | MA7107 / MA7207 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Mathematical Control Theory |
| **Learning Mode** | Lectures |
| **Learning Objectives** | The objective of the course is to train student about the mathematical principles of control theory to analyze and design control systems. |
| **Course Description** | The course is intended to discuss about important mathematical properties of control systems and enables students to solve some optimal control problems. |
| **Course Content** | Control systems and Mathematical modeling, classification of control systems, finite dimensional Deterministic linear control systems, transfer function, state-space representation, computation of transition matrix and solution of linear system, controllability and observability for linear dynamical systems, duality theorem, stability, Liapunov's method, Routh criterion, Nyquist criterion, stabilizability, multivariable system, discrete system, optimal control problem, linear systems with quadratic cost, introduction to calculus of variations and maximum principle. Introduction to infinite dimensional control systems. |
| **Learning Outcome** | By the end of the course, students will be able to describe the fundamental components of control systems, including controllability, observability and stability. They should be able to explain how these components interact to achieve desired system behavior. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. Zabczyk Jerzy , Mathematical Control Theory: An Introduction, Series: Modern Birkhäuser Classics, 1st ed. 1992. 2nd, corr. printing 1995. Reprint, 2008.
2. R.G. Cameron and S. Barnett, Introduction to Mathematical Control Theory, Oxford Univ Press, 1990.

**Reference Books:**

1. Eduardo D. Sontag, Mathematical Control Theory Deterministic Finite Dimensional Systems, Series: Texts in Applied Mathematics , Vol. 6, Springer, 1998.
2. D. Subbaram Naidu, Optimal Control Systems, Series: Electrical Engineering Series Volume: 2, CRC Press, 2002.

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| **Course Number** | MA7108 / MA7208 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Probability Theory and Statistical Inference |
| **Learning Mode** | Lectures |
| **Learning Objectives** | This particular course on Probability Theory and Statistical Inference is intended for postgraduate students. One of the major aims of this course is to provide students fundamental structural concepts of various probability models. It also provides important aspects of statistical inference problems. |
| **Course Description** | Main motivation of this course is to describe and illustrate fundamental concepts of probability theory which are important in formulating statistical inference problems. |
| **Course Content** | Algebra of sets, probability measure, Random variables, Standard discrete and continuous distributions, Distribution of functions of random variables, Expectation, Correlation, Moment generating functions and their properties, Convergence of random variables, Characteristic, Laws of large numbers, Limit theorems. Exponential families, Sufficiency, Completeness, Basu's Theorem, Invariance and maximal invariant statistic. Unbiased estimation, maximum likelihood estimation, method of moments, Bayesian methods, Minimax and admissible estimators, Interval estimation, Equivariance principle, Neyman-Pearson theory, Most powerful, UMP Test, Unbiased Test, Monotone likelihood ratio property, Likelihood ratio tests. |
| **Learning Outcome** | Students will learn probabilistic and structural properties of various probability distributions along with important concepts of statistical theory. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. G. Casella, and R.L. Berger, Statistical inference, Second Edition, Wadsworth, Belmont CA, 2001.
2. V.K., Rohatgi and Md. Ehsanes Saleh, An introduction to probability and statistics, Second Edition, Wiley India, 2009.

**Reference Books:**

1. E. L. Lehmann and G. Casella, Theory of point estimation, Second Edition, Springer (India), 2003.
2. K.L. Chung, A course in probability theory, Second Edition, Academic Press, 2000

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| **Course Number** | MA7109 / MA7209 |
| **Course Credit**  **(L-T-P-C)** | 3-0-0-3 |
| **Course Title** | Topology |
| **Learning Mode** | Lectures |
| **Learning Objectives** | Same as learning outcome |
| **Course Description** | It is an advanced course on Algebraic topology. |
| **Course Content** | Quotient topology, Topological groups, Group Actions, Orbit spaces.  Homotopic maps, Construction of the fundamental group, Fundamental group of circle, Homotopy type, Covering spaces, Borsuk-Ulam and Ham-sandwich theorems, A lifting criterion, Seifert-van kampen theorem, Brouwer fixed point theorem and other applications  Polyhedra, PL maps, PL manifolds, Cell complexes, Subdivisions, Simplicial complexes, Simplicial maps, Triangulations, Derived subdivisions, Pseudomanifolds, Abstract simplicial complexes, isomorphism.  Orientation of complexes, Chains, Cycles and boundaries, Homology groups, Euler-Poincare formula, Barycentric subdivision, Simplicial approximation, Induced homomorphism, Degree and Lefschetz number fixed-point theorem. |
| **Learning Outcome** | At the end of this course, students will learn to:  -compute first homotopy groups (fundamental groups) of several important topological spaces such as spheres, torus, etc., by applying the Seifert-Van Kampen theorem.  -compute the simplicial homology groups of several topological spaces. |
| **Assessment Method** | Quiz /Assignment/ Project / MSE / ESE |

**Text Books:**

1. C. A. Kosniowski, First course in Algebraic Topology, Cambridge Univ. Press, 2008.
2. C. P. Rourke, and B. J. Sanderson, Introduction to Piecewise-Linear Topology, Springer-Verlag, 1982.

**Reference Books:**

1. J. R. Munkres, Elements of Algebraic Topology, The Benjamin Cummings Pub., Co., 1984.
2. M. A. Armstrong, Basic Topology, Springer (India), 2004.
3. K. Janich, Topology, Springer-Verlag (UTM), 1984.